ON THE SHAPE OF THE EARTH

by Leonhard Euler

Article 32 in Eneström's index
Anmerckungen über die Zeitungen, St. Petersburg 1738
[*Opera Omnia*, III_2, 325–346]

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(draft)

Part 27

St. Petersburg, April 3, 1738

After the absurd beliefs of the Ancients concerning the shape of the Earth had been sufficiently disproved, not only by the philosophers, but by actual travels, it appeared to be a settled matter that the Earth must be perfectly round like a ball. This view was supported not only by the various voyages which completely circled the Earth; but in addition the shape of the Earth's shadow, which was seen during lunar eclipses, appeared to confirm it; not to mention other grounds which besides these were cited to support the same conclusion. Despite all this, however, this opinion began to be brought into question during the previous century, as it became clear that all the evidence which had been put forward did not demonstrate perfect roundness, but only a shape which was approximately round. Thus the question arose whether the Earth was exactly like a perfectly round ball, or only approached this shape. For at least this much is and remains certain, that if the Earth were not as perfectly round as a ball, nevertheless the difference must be very small and barely observable. Since then, therefore, there has been a particular effort to decide this question, not only by a variety of observations, but also through deep reflections. It was considered that this question was of no small importance, in part because of the ensuing benefit to Geography, and in part in view of the resulting progress in natural science. To this end, as is well known from the newspapers, famous mathematicians and astronomers were sent out at great expense by the King of France some years ago, some to Peru in America, others to Swedish Lappland. The latter group, on their return to Paris, claimed in fact to have discovered the true shape of the Earth. Since in the public papers there have been several reports on this discovery, we hope to do most of our readers no small favor, if we set forth everything which concerns the question itself, as well as its resolution, as briefly and clearly as possible.

Those who denied the perfect roundness of the terrestial globe have up to now been divided into two completely antagonistic parties, of which the one party maintained that the Earth had a flatter curvature near the *Pole*, and that its shape was similar to that of an orange. The other party believed, on the other hand, that the figure of the Earth was much more extended near the *Pole*, and might be compared with a melon or lime, which two opinions are thus diametrically opposed to one another. Both parties sought to substantiate their views by powerful demonstrations. Those who assigned to the Earth a flatter curvature based themselves primarily on the basic principles of motion, and also brought into account the observations which had been made with the *pendulum* at various locations on the Earth. The other party, however, which compared the Earth with a melon, appealed above all to experience, on the strength of which they believed they had determined by various observations that the degree along a meridian was smaller near the *Pole* than near the *Equator*, which would indeed have been an incontestable demonstration of this view, provided that the observations in questions had been completely correct; in what follows, we will consider this point in more detail. But the French mathematicians who were lately in Sweden now maintain the opposite, and claim to have found with the greatest precision that the degree along a meridian always increases, the nearer one gets to the *Pole*. If now no further doubt remains about this discovery, then it follows quite certainly that the Earth is similar in shape to an orange, and therefore must be thicker under the *Equator* than through the *Poles*. It appears, however, that these observations can be trusted so much more securely, in that the French mathematicians referred to not only were provided with the best and most excellent instruments, but also applied them with every imaginable diligence, as we may expect to see from the detailed report of the whole expedition, which should soon be made public. And it serves not a little for further confirmation, that the Frenchmen had previously all adhered to the other opinion, zealously defending it against the English, all of whom asserted the view now finally found to be the correct one; from which it follows that not the slightest fault can be found with these observations which have been made; inasmuch as the Frenchmen would never have abandoned the view which they had previously upheld, and defended so vehemently, if they had not been most clearly and emphatically convinced of its incorrectness, and simultaneously of the truth of the contrary view. The complete findings concerning the shape of the Earth are anticipated following the return of the French expedition which was sent to America; their observations must be added to and compared with those which were made in Lappland. For although it is now perfectly clear that the Earth is thicker under the *Equator* than through the *Poles*, and resembles an orange; nevertheless the actual ratio between the *Axis* of the Earth which goes from one *Pole* to the other and the *Diameter* of the *Equator* is not yet determined. For this purpose it was found necessary, from the very beginning of this undertaking of the Royal Academy of Sciences, to carry out the most exact observations both under the *Equator* and near a *Pole*, to which end also both intended expeditions to Peru and Lappland were undertaken. Meanwhile, it is remarkable that precisely this flattened curvature, similar, that is, to an orange, was originally set forth purely on the basis of theory; and that the opposing view cannot exist with theory and the rules of motion and of Nature: whence our natural science would have been placed under no small disadvantage, if the experience itself had come out in opposition; now, however, through the agreement of Nature with the theory, the certainty of our knowledge in natural matters will be rather clearly brought to the light. In the following continuations of this discussion, however, we will try to show in more detail, on what basis the shape of the Earth, purely on the basis of reason, without having to consult experience, can be concluded and determined. Subsequently, we will describe those operations and observations which were required, in order to determine the true shape of the Earth purely through experience, so that the basis of the observations actually carried out to this end can be understood all the more clearly. Before that, however, it will be necessary to explain more clearly the true and intrinsic difference between the two figures which have been attributed to the Earth, and to point out adequately what kind of properties the Earth would have, according to the respective views, so that one can then decide which agrees most exactly with experience.

E.

Part 28

St. Petersburg, April 6, 1738

Continuation of the Shape of the Earth

In order to be able to imagine more clearly the nature of the Earth, whether it should have an extended or a flattened curvature, let us first think of it as perfectly round, and investigate the properties that the Earth would have in this case. Here it is to be noted in the first place that the mountains, valleys, and other irregularities of the Earth's surface need not be taken into account, but that the Earth must be conceived as though it were everywhere surrounded and covered with water, and thus had an even and smooth surface all around. For when it is a question of the shape of the Earth, one does not inquire how many mountains and valleys there are on its surface, but one requires rather the investigation of its true figure, supposing that all the land were transformed into water. If we now, therefore, bring the Earth into contemplation, as though it were perfectly round, then it is clear to begin with that all the points on its surface would lie at the same distance from its center, and consequently the axis which goes from one *Pole* to the other would have to be precisely equal to the diameter of the *Equator*. From this it follows further that the weights of bodies on the surface of the Earth would have to be the same wherever they were located, and that they must be directed toward the center. This is confirmed by the nature of balance or *equilibrium*, according to which a fluid body which stands in *equilibrium* cannot press in any direction other than that which is plumb or *perpendicular* to the surface. In the case of a spherical surface, however, all the lines drawn to it from the center must meet it *perpendicularly*, and hence, on the surface of the Earth, if this were a perfect sphere, the direction of gravity would everywhere go straight to the center. For bodies acquire through gravity a power of pressing, such that the direction according to which this pressure occurs goes perpendicularly to the *horizon* or the surface of the water; and a heavy body, when it is not supported, falls straight down in this direction. Thus every line which falls *perpendicularly* onto the surface of the water is called "plumb", since this is the direction shown by a plumb-line. If therefore the Earth were perfectly round, then all vertical lines would go through the center of the Earth, and there would all come together. On the other hand, if the shape of the Earth were otherwise constituted, then it is immediately clear that not all verticals, that is to say, such lines as fall *perpendicularly* to the surface of the water, would be directed through the center of the Earth. This is because no other figure except a sphere has the property that all lines *perpendicular* to its surface come together in one point. Now whether the Earth is perfectly round or not cannot be determined from this property, since it is not possible to investigate, inside the Earth, whether all vertical lines meet together at its center or not. The second property that the Earth would have, if it were perfectly round, would be that a given body would have everywhere on its surface the same weight, and would press downward with the same force. For if one imagines different canals, having the same width, leading from the surface of the Earth to its center, and supposes these canals filled with water, then the weight of the water in all these canals must be the same. For if the water in one canal pressed more strongly on the center than another, then the greater pressure would overpower the lesser and the water in the one canal would sink down while in the other it would be driven higher, until a balance was established; in which case the Earth would have to take on another shape. It follows that if the shape of the Earth is spherical, and consequently these hypothetical canals have equal lengths, then the weight of a body on the surface of the Earth must be everywhere the same; since the weight of the water in these different canals, if they are all the same size, could not be all the same, unless gravity itself were the same everywhere. From this it is easy to see what the nature of gravity on the Earth would have to be, if its shape were not spherical, but either lengthened or flattened. Let us for example stipulate that the shape of the Earth is lengthened, and similar to a melon, or that the axis of the Earth that goes from one *Pole* to the other is greater than the *diameter* of the *Equator*; in which case, therefore, the *Poles* would lie at a greater distance from the center of the Earth than the *Equator*. If we then imagine two canals of the same width, of which one reaches from a *Pole*, the other from the *Equator* into the center of the Earth, then the canal which goes from the *Pole* to the center would hold more water in itself than the other, which has been made from the *Equator* into the center, because the former is longer than the latter. But now because there is more water in the longer canal than in the shorter, therefore an equal *quantity* of water in the longer canal must be lighter, and not press as strongly towards the center, as the same amount of water in the shorter canal. Consequently, gravity under the *Equator* must be greater than under the *Poles*: that is, if the Earth were similar to a melon in shape, then a body by virtue of its weight would press down with a greater force under the *Equator* than if the same body were displaced to one of the *Poles*. If the Earth had this shape, then, a body would have to be heaviest under the *Equator*, and become lighter, the closer it was brought to the *Pole*. In this way, a real and intrinsic difference between the extended and the spherical shape appears, inasmuch as in the case of a perfectly round shape a body would have to have the same weight anywhere on the surface of the Earth. But just as, in the case of the extended shape of the Earth, the greatest weight would occur under the *Equator*, the least under the *Poles*, so, on the contrary, if the Earth were taken to be flattened, or similar to an orange, the weight under the *Equator* would have to be less than that under the *Poles*. For if we once again imagine two equally wide canals, the one reaching from a *Pole* into the center, the other from the *Equator*, then the former will be shorter than the latter, and consequently will contain less water. But because, in view of the equilibrium, the water in both canals must press equally strongly on the center, and therefore the greater *quantity* in the longer canal does not weigh more than the lesser *quantity* in the shorter, therefore the force of gravity must necessarily be greater in the shorter canal than in the longer; that is, the force of gravity must be greatest at the *Poles*, and least under the *Equator*. In this way, then, arises a reliable way of determining through direct experience whether the Earth is perfectly round like a ball, or extended like a melon, or flattened like an orange. If one in other words investigates through accurate observations whether the force of gravity is everywhere around the globe equally great, or whether it is greater, either under the *Equator* or against the *Poles*. In what way, however, one can determine everywhere most precisely the true magnitude of gravity, or of that power which drives bodies downward, will be clarified in detail in the following pages.

E.

Part 29

St. Petersburg, April 10, 1738

Continuation of the previous material

That all bodies on our Earth are heavy, or press downward, is by now a settled matter with all natural philosophers; for although wood in water and vapors in the air climb upward, the cause of that is nevertheless not to be ascribed to a natural levity, but rather to the greater weight of water and air; which is sufficiently demonstrated by the *experiment* that in an empty space, such as one produces by means of an air pump, the lightest feather falls as swiftly downward as gold. To be sure, the cause of this heaviness-producing force is still a matter of great controversy among the learned; so much is nevertheless certain, that this force operates on all portions of matter, and either pushes or pulls them downward. Thus, the more matter a body contains, the more strongly it pushes downward; and for this reason it is customary to determine the amount of matter in a body by means of the magnitude of this pressure: but the pressure becomes perceptible through the weight, which is nothing but the power with which a body strives to fall downward. For this purpose, scales have been invented, by means of which the weight of a given body may be determined. But it can't be determined by weighing whether a body has one and the same weight at different places on the Earth, and presses downward with the same force; for given that the force of gravity decreases or increases, then the weights used with the scales would become just as much lighter or heavier, so that a body would maintain everywhere the same weight according to the scales, independently of any variation in the weight-producing force. Although we have reported in the previous pages about the variation of gravity on the Earth, in case that Earth were either extended or flattened, nevertheless the usual scales cannot be used to investigate this variation; it is necessary, rather, to make use of a completely different test, one which indicates, not the weight of a body, but rather the gravitational force in and of itself. Such a test can now be derived from the speed of fall, and from fall itself; for through the force of gravity a body becomes capable of falling during a certain time through a certain height; and this height must always be the same, no matter how great or small the body may be, provided that the gravitational force remains the same. This must be understood here to refer to fall in airless space, in which we have already remarked that all bodies fall equally fast. If on the other hand the gravitational force itself should become either greater or less, then a body in the same time would fall through either a greater or a lesser height. Thus it has been found that a body which begins to fall, in one second falls about 15 feet: if then precise observations could be undertaken as to how far a body fell in the time of one second, both under the *Equator* and at the *Poles*, then one could pretty well decide the question concerning the figure of the Earth. Thus, if one found that a body, under the *Equator* and the *Poles*, fell through the same height in one second, then one could conclude that the Earth were spherical; if however it should happen that the height from which a body fell in one second was greater under the *Equator* than under the *Poles*, then it would follow that the gravitational force was greater under the *Equator* than under the *Poles*, in which case the Earth would have to be extended in shape like a melon. On the contrary, however, one would have to assign to the Earth a flattened shape like an orange, if a body under the *Poles* fell more swiftly than under the *Equator*. But however easily the present question could be decided in this way, nevertheless it is correspondingly difficult to measure precisely the height through which a heavy body falls in one second; since the fall happens so quickly that it is almost impossible to determine correctly the point which the body in its fall reaches after one second; for which reason in this exercise one makes use of other experiments, which can be made with greater *accuracy*, and which one can trust with greater confidence. To this end, namely, *pendula* will be used, made by hanging a ball made out of lead or some other heavy material on a thread. Because of gravity, such a *pendulum* hangs vertically; if it is then pushed out of this state, then swings or oscillations arise, which continue until the motion is completely checked by resistance. These swings or *oscillations* occur now in a definite time, which *depends* partly on the length of the *pendulum* and partly also on the gravitational force, so that, if the length remains the same, the *pendulum* swings with so many more beats in a certain time, the greater the force of gravity is. But if that force remains unchanged, then there will be as many more beats, the shorter the thread becomes. Consequently, if equally long *pendula* make equally many *oscillations* during the same time everywhere on the earth, then the gravitational force is everywhere the same, and therefore the figure of the Earth is spherical. But if the number of *oscillations* which identical *pendula complete* in the same time is less or greater under the *Equator* than under the *Poles*, then one would have to conclude that the Earth in the first case would be similar to an orange in shape, but in the other case similar to a melon. But in regions which are roughly halfway between the *Equator* and a *Pole*, it has been found that a *pendulum* which is 3 feet 2 inches, in Rheinland measurement, measures a second accurately in its beats. If now a *pendulum* of this length were brought to a place where the gravitational force were less, then it would go more slowly, and therefore would have to be made shorter, in order to beat a second accurately; on the other hand, at places where the gravitational force is greater, then this very *pendulum*, in order to beat seconds, would have to be made longer. From this it becomes clear in what way the length of a *pendulum* which marks seconds by its beats should be adjusted everywhere, for a given shape of the Earth. Namely, in case the Earth is spherical, then this length would be found to be the same everywhere. But if the Earth were extended like a melon, then the lengths of those *pendula* would have to be greater at the *Equator*, but less at the *Poles*, because in this case the gravitational force would be greater under the *Equator* than under the *Poles*. But if the shape of the Earth were to be flattened, or like an orange, then the lengths of these *pendula* which we have been thinking about, which are supposed to beat seconds accurately, would have to be made shorter at the *Equator*, longer at the *Poles*. Now, very careful experiments have been carried out with the length of such a *pendulum*, which indicates seconds accurately by its beats, at different places on the Earth, by means of which it has been quite clearly shown that the nearer one comes to the *Equator*, the shorter the *pendulum* has to be made. And when a few years ago at Archangel, at the direction of the local Academy, the length of such a *pendulum* was determined, it was found to be still greater. It follows from this therefore quite certainly, that the shape of the Earth is not extended, but rather flattened and like an orange.

E.

Part 30

St. Petersburg, April 13, 1738

Continuation of the shape of the Earth

Although in the foregoing pages we have now already definitely decided the question of the shape of the Earth, and have adequately shown by means of the observations carried out with *pendula* that the Earth is thicker under the *Equator* than between the *Poles*, or that it could most appropriately be compared with the shape of an orange: it is nevertheless helpful to carry this material further, and to strengthen it with further demonstrations. For since the proof we have presented is grounded in the operation of gravity and the properties of *pendula* which result from it, it is possible that this proof might not be completely accepted by those who do not sufficiently grasp these truths. Besides that, all that we have been able to show is that the Earth is thicker under the *Equator* than between the *Poles*; for a complete knowledge of the shape of the Earth, it is required that one knows precisely how much thicker the Earth is under the *Equator* than between the *Poles*, or that one can express the ratio between the *diameter* of the *Equator* and the Earth's axis which goes from one *Pole* to the other. In order now to investigate this, further observations will be required, through which one can determine at every place the curvature of the surface of the Earth; these observations, which are also more striking, will therefore serve in addition to confirm more strongly the previous proof. It is well known to anyone who has travelled on the sea that one cannot there see to a great distance, even when one uses the best telescope; and consequently this deficiency can hardly be ascribed to the weakness of our sight. For when a ship approaches from a distance, one sees at first only the upper top of the mast as it were sticking out of the water; but nothing of the ship itself can be perceived, although it is surely not at a greater distance from us than the mast. But the closer the ship comes, the more of it can be seen, until finally the whole ship is clearly in sight. Now this could not possibly happen if the sea were perfectly flat, in which case one would be able to see even the most distant objects that float on the sea, and so in this case, as soon as one could see the mast, then at the same time the whole ship would be in sight. Since this is not what happens, it is clear that the sea is not perfectly flat, but must have a notable curvature, because of which objects which are at too great a distance are hidden from us. For then the sea forms as it were a hill between us and the ship approaching from a distance, and therefore brings about the effect that we see the upper top of the mast before the whole ship comes into our sight. From this, everyone will easily grasp that the more pronounced the curvature of the Earth is, the distance over which one can see must be correspondingly smaller. But now if one takes into consideration the distance to which one can see at a given height over the sea, and applies some simple rules from geometry, then one can in fact compute the *quantity* of the curvature at the given place, which amounts to finding the size of a ball which is curved the same as the surface of the sea. If one has found in this way the amount of curvature of the sea, or of the Earth, then one knows immediately how far one degree extends along it, since one knows the size of a similarly curved ball, and that there are 360 degrees around the whole circumference. But since the surface of a larger ball is less curved than that of a smaller one, it follows that a degree on the Earth must be so much the smaller, the more strongly curved its surface is. Now it is customary, in relation to our objective of determining the shape of the Earth, to look particularly at the *meridia*, which are the lines which one imagines to be drawn along the surface of the Earth directly from South to North. Thus, all these *meridia* come together at the *Poles*, and are cut by the *Equator* into equal pieces, so that one part of a *meridian*, which is contained between the *Equator* and a *Pole*, represents *accurately* the fourth part of the whole circumference of the Earth. But since the entire circumference of the Earth is divided into 360 degrees, then such a part of a *meridian*, situated between the *Equator* and a *Pole*, must contain just 90 degrees. Consequently, whatever the figure of the Earth may be, there will always be 90 degrees from the *Equator* to either *Pole*, and therefore 180 degrees from one *Pole* to the other, given that one always proceeds directly from South to North; after the figure of the Earth has been specified, however, these 90 degrees from the *Equator* to a *Pole* may turn out to be either equal or unequal among themselves. Now everyone will easily see that if the Earth should be perfectly spherical, then every degree along a *meridian* must be the same as every other; for in such a case the *meridia* are perfect circles, and therefore have everywhere the same curvature or bending, which implies the equality of the degrees. But if the shape of the Earth is taken to be extended or similar to a melon, then it is clear that the *meridia* will not be circles, but rather *ovals*, having an extended curvature; such a *meridian* would therefore have a greater bending or curvature next to the *Poles* than under the *Equator*; and consequently the degree along such a *meridian* would have to be greater under the *Equator* than at the *Poles*. If on the other hand the Earth had a flattened curvature, or were similar to an orange, then each *meridian* would have the greatest curvature under the *Equator*, but the least curvature under the *Poles*. For this reason, the degrees along each *meridian* would have to be smallest under the *Equator*, but greater under the *Poles*; and this distinction would be so much the greater, the more the figure of the Earth were flattened, and departed from perfect sphericity. Just as we found the first perceptible distinction, arising from the different figures of the Earth, in the different nature of gravity on the Earth's surface; so the second perceptible distinction rests on the different property of the degrees of a *meridian*; and this serves like the first to find the true figure of the Earth by observation. For if one were to survey most accurately the size of a degree on a *meridian* near to the *Equator* as well as to a *Pole*, then one would find both these degrees either equal to one another, or unequal. In the first case the Earth would consequently have to be perfectly round, but in the other case either elongated or flattened, depending on whether the degree at the *Equator* were greater or smaller than the degree at the *Pole*. Aside from these two primary distinctions, however, there is a third which can be appealed to, in order to determine the figure of the Earth. This third distinction consists now in the size of the degrees along the *Equator*, which in comparison with the degrees along the *meridians* must have a different character, depending on whether the figure of the Earth is either completely round or elongated or similar to an orange. For if the Earth is perfectly round, then all the degrees of a *meridian* would have to be equal, not only among themselves, but also to the degrees of the *Equator*. But if the Earth had an elongated figure like a melon, then a degree along a *meridian*, measured next to the *Equator*, would have to be greater than a degree along the *Equator*. But in the case that the Earth has a flattened figure similar to an orange, then a degree along the *Equator* will have to be greater than a degree along a *meridian*, taken near the *Equator*; so that by this means the true figure of the Earth can be determined just as well as by the previous ones.

E.

Part 31

St. Petersburg, April 17, 1738

Continuation of the shape of the Earth

Since we have shown in the foregoing pages how the true figure of the Earth can be determined by the accurate measurement of the degrees along the *meridians* as well as along the *Equator*, our plan now requires us to describe the various procedures and operations, by means of which one can actually find the size of such degrees. Now this measurement occurs either through astronomical observations, in which one determines the size of a degree on the Earth from the height of stars and other celestial observations; or purely through geometric operations, not referring to the stars or other celestial occurrences. Now to begin with, as far as concerns astronomical observations, it is to be noted that, if one goes along a *meridian* one degree nearer to the *Pole*, the height of the *Pole* also increases by one degree; for if one were to imagine a star in the celestial *Pole* itself, which would consequently have no motion, then this star would be seen *precisely* at the *Zenith* by someone standing at the *Pole* of the Earth, and would therefore stand 90 degrees above the *horizon*. But if this observer were to be located under the *Equator*, then he would see this *Pole*-star just at the *horizon*, and consequently would perceive the *Pole* to have no height. It is accordingly clear from this that, the further one goes from the *Equator* toward the *Pole*, the higher the star at the celestial *Pole* must appear to be; and from this it becomes clear how by observation of the height of the *Pole* one can find out how far from the *Equator* one is located on the Earth. In this way, however, this distance will not be given in the conventional units, but rather in degrees, whose size is still unknown; thus, since here in St. Petersburg the *Pole* stands about 60 degrees high, we know from that that we are 60 degrees distant from the *Equator*, but 30 degrees from the North *Pole*; but what that distance amounts to in miles or versts remains still unknown. But if one were to travel from here directly along a *meridian* to the *Equator* or to the *Pole* and were to measure the way travelled most accurately in feet, then one would know how many feet those 60 degrees to the *Equator* amounted to, as well as those 30 degrees to the *Pole*. Now although such a world-encompassing survey cannot actually be brought about; nevertheless one sees from this how by means of astronomical observations and simultaneously precise measurement on the Earth the size of the degrees along a *meridian* can be found; so if for example I travel directly North from here, where the height of the *Pole* has been found to be $59°,57'$, until I find the *Pole*'s height to be $60°,57'$, then the distance travelled along the St. Petersburg *meridian* amounts to precisely one degree; and if I now measure the distance travelled in feet, then I know how many feet go into a degree along a *meridian* under a *Pole*-height of roughly 60 degrees. If one carries out similar measurements of a degree along a *meridian* at various *Pole*-heights, then in this way one will find whether all these degrees are equal to one another or not; and if an inequality is perceived, whether the degree near the *Equator* is greater or less than than the degree which is measured nearer to the *Pole*. From this sort of operations one would accordingly be able to determine fairly accurately the true figure of the Earth; for if one were to find all the degrees equal to one another, it would follow that the Earth had a perfectly round *spherical* figure; but if the degrees lying near to the *Equator* were greater than those measured nearer to the *Pole*, then the figure of the Earth would have to be elongated and similar to a melon. On the other hand, we would assign to the Earth a flattened shape similar to an orange, if the degrees toward the *Pole* were found to be greater than those toward the *Equator*, which in the foregoing remarks has been sufficiently shown. It is mostly in this way that, up to now, people have gone about trying to determine the figure of the Earth, and past as well as contemporary mathematicians have measured the size of a degree along a *meridian* under various heights of the *Pole*. Among all these operations, however, that one is particularly noteworthy, in which some time ago the French mathematicians drew a *meridian* all the way through France, and surveyed most diligently all the degrees along it. As a result of this the French believed that they had determined with certainty that the degrees in the southern part of France were appreciably smaller[1)] than in the northern part, and consequently they have on this basis up to now assigned an elongated shape to the Earth, which they have also most zealously defended against those who for other reasons would not accept this figure. But now that likewise French mathematicians in the previous year in Lappland have surveyed a degree along the *meridian* most precisely, they have found very clearly that this degree measured in Lappland was greater than those which had previously been measured in France. For this reason they were compelled to abandon their previous opinion, and to give assent to the other, which stipulated the figure of the Earth to be flattened and similar to an orange. In this connection, many will without doubt be astonished, both that observations, though carried out with the greatest care, could have confirmed an incorrect view, and that for such a long time the correct opinion could have been cast into doubt, even though sufficiently supported by observations made with the *pendulum*. However we will soon clearly show that such astronomical measurements of degrees, if they are not made with the greatest *accuracy* and at places far distant from each other, are insufficient by far to settle the question of the figure of the Earth. For even if an observation of the height of a star is carried out as carefully as it is possible to be, nevertheless the instruments cannot be so accurately constructed and calibrated that it is accurate to less than 5 seconds. Now since two such observations of the height of the *Pole* are required for the measurement of a degree, a double error can easily occur, and thus a degree of either 10 seconds too great or too small can result. But a degree contains roughly 343 000 French feet, and consequently 10 seconds amount to about 1000 feet, whence it is clear that one cannot by this method measure a degree to an accuracy of more than 1000 feet. If therefore the figure of the Earth does not deviate very strongly from the round, which on other grounds, through which previously the perfect roundness of

[1)] German *kleiner*. BURCKHARDT, who edited this article for the *Opera Omnia*, observed that we should here read *greater* (*grösser*). *Tr.*

the Earth was shown, is sufficiently clear, then the difference between two degrees along a *meridian*, which are not too far apart from one another, cannot possibly be great enough that one could perceive it in view of the above-mentioned error of 1000 feet. And for this reason the figure of the Earth similar to an orange was quite easy to defend against the observations made in France, according to which the greatest difference between the northern and southern degree amounted only to about 1000 feet, and for this reason should have been attributed much more to the unavoidable errors of observation than to an elongated figure of the Earth.

E.

Part 32

St. Petersburg, April 20, 1738

Final continuation of the shape of the Earth

Now that we have shown in the last issue how by means of celestial observations the degrees along a *meridian* can be measured, the inequality of which serves to determine the figure of the Earth; it still remains to show how, likewise, the size of a degree along the *Equator* can be found. But since, as long as one remains at the *Equator*, no variation in the height of the *Pole* can be perceived, thus this type of observation, which undertakes to find the midday height of stars, cannot be useful for this purpose; one must rather make use of such observations as one customarily uses to find the *longitude* on the Earth's surface. But this is done either by means of eclipses of the Moon, or by means of the eclipses of the moons of Jupiter, inasmuch as the time when such a moon either enters the shadow of Jupiter or reappears from it can be observed most precisely. For if such an entry or reappearance is observed simultaneously at two places whose longitude differs by a degree, it will be found that the times at which this occurs at both places will *differ* by four minutes, assuming, to be sure, that at both places the clocks were previously set precisely according to the Sun. If therefore at two places lying along the *Equator*, whose *geometric distance* has been measured, and consequently is known, one observes an eclipse of the Moon, or an *entry* or *reappearance* of a moon of Jupiter, and notes the difference in the respectively observed times, then one will find how many degrees and parts of a degree the distance between those two places along the *Equator* amounts to; and consequently one will determine from this the size of a degree along the *Equator*. Now if one has also determined the size of a degree along a *meridian* near to the *Equator*, then by comparison of these two degrees one will easily find out whether the figure of the Earth is round, or elongated, or orange-shaped; for if both degrees are equal to one another, then it follows that the Earth must be round, whereas if the degree of the *Equator* is smaller than the other, then the figure of the Earth must be elongated. On the other hand, however, if the degree of the *Equator* is found to be greater than the degree of the *meridian*, then it follows inescapably that the shape of the Earth must be flattened and similar to an orange. In order to undertake this operation and measurement of two such degrees along the *Equator* and along a *meridian*, some astronomers and *Geometers* were sent some years ago by the Royal Academy of Sciences in Paris to America in the province of Peru, from whom presumably the perfect confirmation will shortly be obtained that the Earth is similar to an orange in its figure; inasmuch as this figure has already been confirmed by two tests, namely the *pendula*, and the inequality of the degrees along a *meridian*. But just as we have noted above that those observations which are made to find the height of the *Pole* are still always subject to some errors, so that one can hardly be more certain of the size of a degree along a *meridian* than up to 10 seconds, which amount to 1000 feet; in the same way these observations, through which it is customary to find the difference in longitude of two places along the *Equator*, are subject to an even greater uncertainty. For since four minutes difference in time already amounts to a whole degree on the Earth, it follows that an error of only a single second in the time, which it is hardly possible to avoid, amounts to 1500 feet, which is so great that one could derive from it any desired figure whatever for the Earth. From all this it is now sufficiently clear, that all measurements of the degrees, both along a *meridian* as well as along the *Equator*, are far from sufficient to determine the figure of the Earth, and that, in order to decide this question, the observations which have been undertaken with the *pendulum* should be given far greater weight than all these measurements. Perhaps however it might be possible to bring it about purely through geometric observations alone, that one could determine the size of a degree both along the *Equator* and along a *meridian* with greater precision; which could be accomplished either by a complete voyage around the Earth, or by other geometric instruments prepared with the greatest diligence and applied to the work in various ways.

In order not to linger over this material too long, let us in conclusion present a few other considerations of a completely different nature, through which the flattened round figure of the Earth, though not fully proved, is nevertheless sufficiently established. The first is a likewise orange-shaped figure of the planet Jupiter, in which the axis which goes from one *Pole* to the other is a tenth part shorter than the *diameter* of its *Equator*, as has been confirmed through many observations. If then the same phenomenon were observed in connection with all the other planets, then one could so much the less have any doubt that the Earth also would have a similar figure. But since other circumstances don't allow such observations to be made for the remaining planets, then at least this figure of Jupiter would give some support for the belief in the flattened curvature of the Earth, if one were not led to it in any other way. But if one leaves the decision of this question purely to reason alone, then one finds that, according to the laws of Nature and motion, not only the Earth and Jupiter, but also all other planets which rotate about their axes must necessarily have such a flattened curvature. But since this conclusion has been confirmed for Jupiter through observations, it is so much the less to be doubted for all the other planets. But the way in which motion about an axis necessarily produces such a figure can be made fairly clearly evident to anyone. If one namely imagines such a planet, before it receives a motion about its axis, then one will easily see that this planet, if partially composed of fluid material, must take on a perfectly round figure. The cause of this rests on the nature of gravity, because of which in all planets as well as on the Earth all bodies are driven inwards toward the center. If one then takes well into consideration the action of gravity according to the basic laws of Nature, then one will find that each planet, in the circumstances we are supposing, must be perfectly round, and further that on the surface of the planet gravity must be everywhere the same. Now if such a planet receives a motion about its axis, then everyone will easily see that because of the rotation first of all the gravity along the *Equator* will be lessened, while under the *Poles* it must remain unchanged. Accordingly, from this very rotation the fluid parts must be driven toward the *Equator*, where the motion is greatest, whereby the planet will lose its perfectly round figure, and become thicker under the *Equator* than between the *Poles*. How incomparably now this *raisonnement* agrees with the measured figure of the Earth and with the variation of gravity on it, everyone will recognize with amazement, and so much the less will be able to doubt either the figure of the Earth itself, or the basic laws of natural Science, still less the motion of the Earth. Those however who are able apply Mathematics to this question, will be even more convinced of all these things, if they calculate the actual proportion between the axis and the *diameter* of the *Equator* from the time in which a planet rotates about its axis, and find this perfectly in agreement with observations.

E.

Parts 103 and 104

St. Petersburg, December 25, 1738

Further reports concerning the true shape of the Earth

In those comments on the shape of the Earth which we have communicated to our readers some time ago, we frequently mentioned in particular the survey of the Earth in Lappland undertaken by the Royal Academy of Sciences in Paris. But since at that time we had received so little comprehensive information concerning the nature of this work, or the discoveries it produced, so we hope now to do no small kindness to most readers, if we briefly describe the true circumstances together with the plan and the outcome of this expedition. For this work seemed to the Paris Academy to be of such great importance that they found it advisable to present the complete description of it at once to the entire world, and to publish it in a special memoir, which has been prepared by Mr. Maupertuis, the Director of the expedition. In this memoir the observations and measurements are communicated as they were made and recorded, without any corrections, so that everyone can see how these agree among themselves and to what extent the question of the figure of the Earth can be decided from them; in contrast, others who have previously undertaken this sort of measurements have not published the observations in and for themselves, but rather with the corrections they deemed it appropriate to make. In addition, these French Academicians have most carefully measured all three angles in each of the triangles that were necessary for their project. For although in any triangle all three angles together must amount to 180 degrees, so that one could content onesself with measuring only two angles, nevertheless it could easily happen that the smallest error committed in the measurement of the two angles could become perceptible in the third. In this whole undertaking, therefore, nothing was omitted which could contribute in the least to the exact determination of a degree along the meridian in the same northern region of Sweden. For in addition to the fact that these people recognized perfectly the difficulties of this work and were sufficiently capable of overcoming them by means of ingenious inventions and arrangements, they were also provided with such expensive and carefully prepared instruments as one would find in a well-appointed observatory. In a word, it is sufficiently clear from all circumstances that the French Academy completely accomplished the goal which they had for this expedition, and consequently determined the true shape of the Earth as exactly as one could ever hope. And when the other expedition, which was sent from France to Peru with the same objective, is completed, then this question will remain once and for all answered and decided. With both of these expensive undertakings, the Crown of France has had in view not only the general usefulness of this question, but also the furtherance of science. We have already in the previous notes referred to the controversies which have taken place among the learned over the last 50 years in relation to the figure of the Earth; among which the interested reader will recall that some assigned to the Earth a curvature flattened at the poles, others an elongated curvature. If therefore this question belonged solely to natural Science, even so if would be of such importance that it would merit above all others to be taken into consideration by the learned; but the correct decision of this question is also linked with very many and important advantages in ordinary life. For even if on the globe and the land- and sea-charts the location of all places by longitude and latitude were determined most exactly, one would still not know the true distance from one place to the other, so long, indeed, as one had not determined the figure of the Earth as well as its size; on which the size of each degree, along the meridians as well as along the parallels, depends. But if we are not able to publish the distance of places, it is easy to apprehend what kind of uncertainty and danger sea-travellers would find themselves in. If the Earth were perfectly round, it would be sufficient for this end to have measured exactly a single degree along a meridian, inasmuch as all other degrees along the meridians would be equal to it, whereas the degrees of latitude along the parallels could easily be determined from it. As long as people assented to this belief, rulers as well as scholars have from time to time occupied themselves with measuring the size of a degree, but the computations of the ancients agree so poorly among themselves that one differs from another by more than a half. But the measurements of the Earth which have been undertaken in more recent times can be relied upon nearly as little as those of the ancients. For even those measurements which have been left behind by Fernel, Snell and Riccioli agree together so badly that for a single degree they differ from one another by about 8000 rods, that is, by nearly a seventh part. Furthermore it was not possible to tell, at that time, which of these measurements merited being preferred to the others. The first measurement from which one could attain to any certainty was carried out in England by the celebrated Norwood, according to which the size of a degree along the meridian was established to be 367 196 English feet or 57 300 French rods, each of which contains 6 feet. But when King Louis XIV. of France directed his Academy to determine the size of the Earth, a work was soon brought forth which greatly excelled all the previous ones. M. Picard, who took this work on himself, found the size of a degree along the meridian to be 57 060 rods; and from all the circumstances one was able to conclude that this measurement must have been very accurate; for which reason the King ordered this work to be continued, and that the meridian be measured through all of France, which was also carried out by M. Cassini, through whom the measurement given by Picard was confirmed. Later Mr. Musschenbrock in Holland undertook a similar measurement, and improved the error committed by his predecessor Snell, determining the size of a degree most exactly to be, according to the French measure, 57 033 rods and 9 inches. The difference between these last measurements is now so small that one could be perfectly satisfied with them, and as a result be able to determine adequately the size of the Earth, were it not that at just this time it began to be doubted whether the Earth were perfectly round. For in that case these measurements, however accurate they might have been, were not sufficiently accurate to determine the size of the terrestial globe. The two different views which arose on this question, and what kind of proofs were brought forth to sustain each of them, we have described in detail in the previous pages; but the difference which could arise from these in travel by sea, if some supposed the Earth to be orange-shaped, while others held it to be melon-shaped, is so great, that their reckonings in a voyage of 100 degrees in longitude would differ from one another by more than two degrees: from which it becomes sufficiently clear that the uncertainty concerning the figure of the Earth is linked with very great danger and that consequently one has had the most important motive to decide this controversy. For although the mariners don't very easily observe this error, coming from an incorrect knowledge of the figure of the Earth, inasmuch as they are subject to so many obstacles to determining correctly the whole path of the voyage; nevertheless it is certain that all discoveries leading to a greater certainty in this most necessary matter will bring little advantage so long as the first error concerning the figure of the Earth is not set aside. For this reason the King of France has instituted both expeditions to the North and South, and commissioned experts from the Academy of Sciences to make most carefully all observations necessary for the correct determination of the true size and figure of the Earth. Now concerning the expedition which was dispatched to Swedish Lappland, they measured most precisely a section at $57°28'$ of the meridian from Torneå to Kittis, whence they concluded that the size of a degree in this northern region was 57 427 rods; so that consequently a degree of the meridian under a height of the Pole of 66 degrees was 377 rods greater than a degree in France under a height of the Pole of 48 degrees. From this it follows therefore incontrovertibly that the Earth must have a curvature flattened toward the Poles, or shaped like an orange, and that consequently the axis of the Earth which is drawn from one Pole to the other must be significantly shorter than the diameter of the Equator. In fact, these observations make this difference even greater than the celebrated NEWTON himself had supposed, who found the axis of the Earth to be only one 240^{th} part shorter than the diameter of the Equator; in contrast, from these newest French observations this difference comes out to be nearly twice as great. NEWTON however should not be blamed on account of this small error: inasmuch as this discrepancy arose from his assumption that the Earth consisted of material which was everywhere equally dense, and he himself on the basis of his theory indicated that, if the inner material of the Earth should be denser than the outer, the difference he proposed should be even greater. If one on this account takes, with good reason, the theory of this great man to be correct, then it follows at once from the now completed French observations that the Earth towards its center consists of a much denser material than toward its surface, a view which has previously seemed very probable to many natural philosophers, but could never be completely settled.

E.